From the calculations, one can see that below the dotted line at which $\phi = 1.0$ in./hr, the rainfall infiltrates into the ground and the rainfall above this line (a total of 4.9 in. in 12 hours) runs off.

Theoretical Infiltration Methods*

Theoretical approaches include the solution of the governing equation of continuity and Darcy's law (Chapter 8) in an unsaturated porous media. The governing equation, presented in more detail in Appendix D.2, takes the form

$$\frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial z}\left[K(\theta)\frac{\partial \psi(\theta)}{\partial z}\right] - \frac{\partial K(\theta)}{\partial z}, \qquad (1.23)$$

*combined statement of mass and momentum conservation

where

θ = volumetric moisture content (0/0),
z = distance below the surface (cm),
$\psi(\theta)$ = capillary suction (pressure) (cm of water),
$K(\theta)$ = unsaturated hydraulic conductivity (cm/s).

Hydraulic conductivity $K(\theta)$ relates velocity and hydraulic gradient in Darcy's law. **Moisture content** θ is defined as the ratio of the volume of water to the total volume of a unit of porous media. For saturated ground water flow, θ equals the **porosity** of the sample n, defined as the ratio of volume of voids to total volume of sample; for unsaturated flow above a water table, $\theta < n$. The **water table** defines the boundary between the unsaturated and saturated zones and is defined by the surface on which the fluid pressure P is exactly atmospheric, or $P = 0$. Hence, the total hydraulic head $\phi = \psi + z$, where $\psi = P/\rho g$, the pressure head.

The value of ψ is greater than zero in the saturated zone below the water table and equals zero at the water table. It follows that ψ is less than zero in the unsaturated zone, reflecting the fact that water is held in soil pores under surface-tension forces. Soil physicists refer to $\psi < 0$ as the tension head or capillary suction head, which can be measured by an instrument called a tensiometer.

To further complicate the analysis of unsaturated flow, the moisture content θ and the hydraulic conductivity K are functions of the capillary suction ψ. Also, it has been observed experimentally that the θ-ψ relationships differ significantly for different types of soil. Figure 1.16 summarizes unsaturated zone parameters and relationships.

* This section may be omitted without loss of continuity in text material.

Bedient, P.B. and Huber, W.C. 1992. Hydrology and floodplain Analysis, 2nd ed. Addison-Wesley publishing Co., Reading, MA.

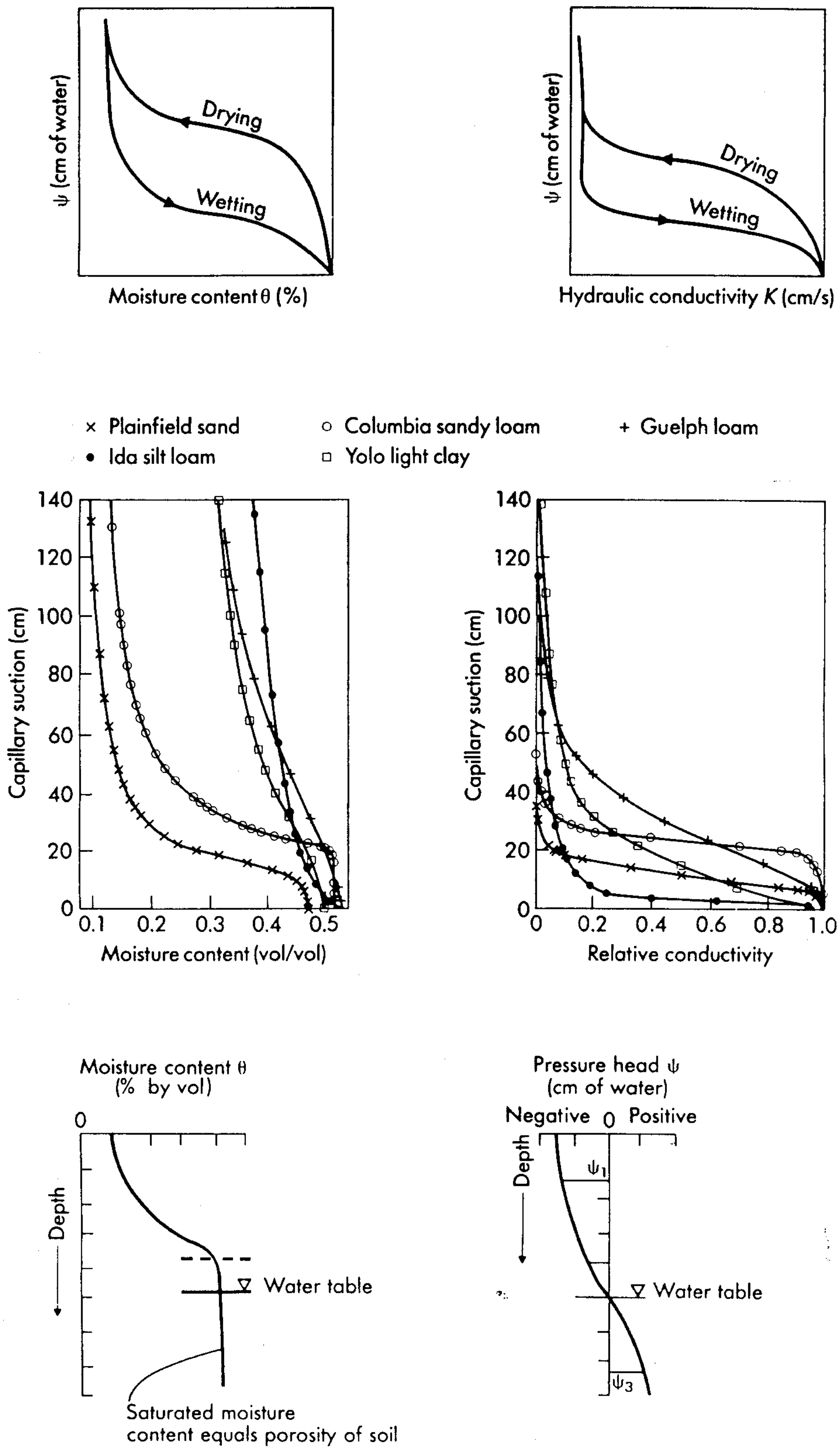

FIGURE 1.16

Typical θ-ψ relationships in the unsaturated zone.

Equation (1.23) is called Richards (1931) equation and is a very difficult partial differential equation. Both numerical and analytical solutions exist for certain special cases. The most difficult part of the procedure is determining the characteristic curves for a soil, which relate unsaturated hydraulic conductivity K and moisture content θ to capillary suction ψ. The characteristic curves reduce to the fundamental hydraulic parameters K and n in the saturated zone and remain as functional relationships in the unsaturated zone.

Philip (1957) solved Eq. (1.23) analytically for the condition of excess water at the surface and given characteristic curves. His coefficients can be predicted in advance from soil properties and do not have to be fitted to field data. However, the more difficult case where the rainfall rate is less than the infiltration capacity cannot be handled by Philip's equation.

One of the most interesting and useful approaches to solving the governing equation was originally advanced by Green and Ampt (1911). In this method, water is assumed to move into dry soil as a sharp wetting front. At the location of the front, the average capillary suction head $\psi = \psi_f$, is used to represent the characteristic curve. The moisture content profile at the moment of surface saturation is shown in Fig. 1.17(a). The area above the moisture profile is the amount of infiltration up to surface saturation F and is represented by the shaded area of depth L in Fig. 1.17(a). Thus, $F = (\theta_s - \theta_i)L = M_d L$, where θ_i is the initial moisture content, θ_s is the saturated moisture content, and $M_d = \theta_s - \theta_i$ the initial moisture deficit.

Darcy's law (Chapter 8) is then used with the unsaturated value for K and can be written

$$q = -K(\theta)\frac{\partial h}{\partial z} \tag{1.24}$$

where

q = Darcy velocity (depth/time),
z = depth below surface (depth),
h = potential or head = $z + \psi$ (depth),
ψ = tension or suction (negative depth),
$K(\theta)$ = unsaturated hydraulic conductivity (depth/time),
θ = volumetric moisture content.

Equation (1.24) is then applied as an approximation to the saturated conditions between the soil surface (subscript "surf") and the wetting front (subscript "wf"), as indicated in Fig. 1.17(b),

$$q = -f \simeq -K_s(h_{\text{surf}} - h_{\text{wf}})/(z_{\text{surf}} - z_{\text{wf}}), \tag{1.25}$$

in which it is assumed that the Darcy velocity (positive upward) at the soil surface equals the downward infiltration rate, $-f$, and the saturated hy-

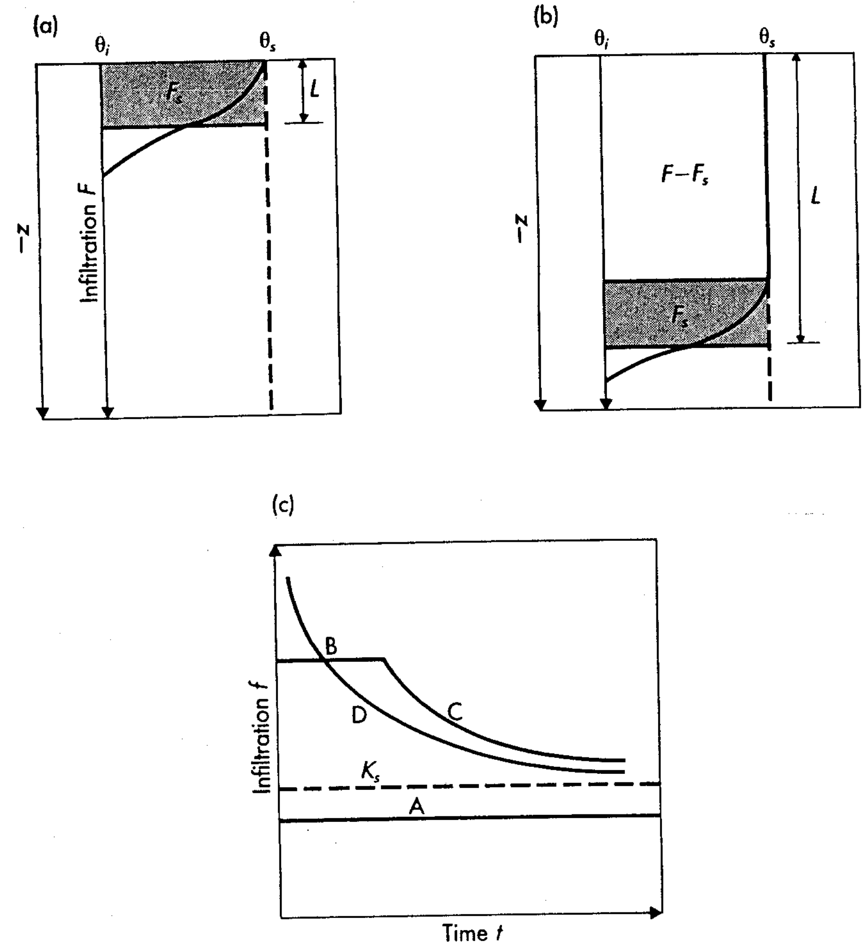

FIGURE 1.17

Moisture and infiltration relations. (a) Moisture profile at moment of surface saturation. (b) Moisture profile at later time. (c) Infiltration behavior under rainfall. (Adapted from Mein and Larson, 1973.)

draulic conductivity, K_s, is used to represent conditions between the surface and the wetting front. The depth to the wetting front is L. Thus, with the coordinate z positive upward, $z_{\text{wf}} = -L$. Using the average tension at the wetting front, ψ_f, we have

$$h_{\text{wf}} = z + \psi \approx -L + \psi_f. \tag{1.26}$$

Noting that $h = 0$ at the surface, Eq. (1.25) becomes

$$-f = -K_s[0 - (-L + \psi_f)]/[0 - (-L)]$$

$$f = K_s(1 - \psi_f/L). \tag{1.27}$$

The volume of infiltration down to the depth L is given by

$$F = L(\theta_s - \theta_i) = L\, M_d. \tag{1.28}$$

Substituting for L in Eq. (1.27) gives the original form of the Green-Ampt equation,

$$f = K_s[1 - (\theta_s - \theta_i)\psi_f/F] = K_s[1 - M_d\psi_f/F]. \quad (1.29)$$

Remembering that ψ_f is negative, Eq. (1.29) indicates that the infiltration rate is a value greater than the saturated hydraulic conductivity—as long as there is sufficient water at the surface for infiltration, as sketched in curves C and D of Fig. 1.17(c). Functionally, the infiltration rate decreases as the cumulative infiltration increases.

As mentioned in the discussion of the Horton equation, the rainfall intensity, i, is often less than the potential infiltration rate given by Eq. (1.29), in which case $f = i$. Mein and Larson (1973) show how Eq. (1.29) can be used to develop the total infiltration curve. At the moment of surface saturation, $f = i$. Let the corresponding volume of infiltration be F_s. With $f = i$, Eq. (1.29) can then be solved for F_s, the volume of infiltration at the time of surface saturation (t_s, the time at which Eq. 1.29 becomes valid),

$$F_s = [(\theta_s - \theta_i)\psi_f]/[1 - i/K_s] = M_d\psi_f/(1 - i/K_s). \quad (1.30)$$

We require $i > K_s$ in Eq. (1.30) and remember that ψ_f is negative. The Green-Ampt infiltration prediction is thus the following:

1. If $i \le K_s$, then $f = i$ (curve A in Fig. 1.17c)
2. If $i > K_s$, then $f = i$ until $F = i\,t_s = F_s$ (Eq. 1.30)
3. Following surface saturation,

$$f = K_s[1 - M_d\psi_f/F] \text{ (Eq. 1.29) for } i > K_s \text{ and } f = i \text{ for } i \le K_s.$$

The combined process is sketched in curves B–C of Fig. 1.17(c). As long as the rainfall intensity is greater than the saturated hydraulic conductivity, the infiltration rate asymptotically approaches K_s as a limiting lower value. Mein and Larson (1973) found excellent agreement between this Green-Ampt method, numerical solutions of Richards's equation, and experimental soils data. If the rainfall rate starts above, drops below, and then again rises above K_s during the infiltration computations, the use of Green-Ampt becomes more complicated, making it necessary to redistribute the moisture in the soil column rather than maintaining the assumption of saturation from the surface down to the wetting front shown in Fig. 1.17(b). The use of Green-Ampt procedures for unsteady rainfall sequences is illustrated by Skaggs and Khaleel (1982).

Equation (1.29) predicts infiltration rate, f, as a function of cumulative infiltration, F, not time. Because $f = dF/dt$, the equation can be converted into a differential equation, the solution of which can be solved

TABLE 1.4

Green-Ampt Infiltration Parameters for Various Soil Texture Classes

SOIL CLASS	POROSITY η	EFFECTIVE POROSITY θ_e	WETTING FRONT SOIL SUCTION HEAD ψ (cm)	HYDRAULIC CONDUCTIVITY K (cm/hr)	SAMPLE SIZE
Sand	0.437	0.417	4.95	11.78	762
	(0.374–0.500)	(0.354–0.480)	(0.97–25.36)		
Loamy sand	0.437	0.401	6.13	2.99	338
	(0.363–0.506)	(0.329–0.473)	(1.35–27.94)		
Sandy loam	0.453	0.412	11.01	1.09	666
	(0.351–0.555)	(0.283–0.541)	(2.67–45.47)		
Loam	0.463	0.434	8.89	0.34	383
	(0.375–0.551)	(0.334–0.534)	(1.33–59.38)		
Silt loam	0.501	0.486	16.68	0.65	1206
	(0.420–0.582)	(0.394–0.578)	(2.92–95.39)		
Sandy clay loam	0.398	0.330	21.85	0.15	498
	(0.332–0.464)	(0.235–0.425)	(4.42–108.0)		
Clay loam	0.464	0.309	20.88	0.10	366
	(0.409–0.519)	(0.279–0.501)	(4.79–91.10)		
Silty clay loam	0.471	0.432	27.30	0.10	689
	(0.418–0.524)	(0.347–0.517)	(5.67–131.50)		
Sandy clay	0.430	0.321	23.90	0.06	45
	(0.370–0.490)	(0.207–0.435)	(4.08–140.2)		
Silty clay	0.479	0.423	29.22	0.05	127
	(0.425–0.533)	(0.334–0.512)	(6.13–139.4)		
Clay	0.475	0.385	31.63	0.03	291
	(0.427–0.523)	(0.269–0.501)	(6.39–156.5)		

The numbers in parentheses below each parameter are one standard deviation around the parameter value given.
Source: Rawls, Brakensiek, and Miller, 1983.

iteratively for $F(t)$ (Chow et al., 1988). Then Eq. (1.29) can be used to determine $f(t)$.

A major advantage of the Green-Ampt model is that, in principle, the necessary parameters, K_s, ψ_f, and $M_d = \theta_s - \theta_i$, can be determined from physical measurements in the soil, rather than empirically as for the Horton parameters. For example, saturated hydraulic conductivity (often loosely called permeability) is tabulated by the U.S. Soil Conservation Service (SCS) for a large number of soils as part of that agency's Soil Properties and Interpretation sheets (available from local SCS offices). An increasing quantity of tension vs. moisture content data (of the type shown in Fig. 1.16) are also available, from which a value of ψ_f can be obtained by integration over the moisture content of interest. For example, several volumes of such information have been assembled for Florida soils (e.g., Carlisle et al., 1981). In practice, the Green-Ampt parameters are often calibrated, especially when used in continuous simulation models.

A useful source of information on Green-Ampt parameters is provided by Rawls et al. (1983), who present data for a large selection of soils from across the U.S. These data are shown in Table 1.4. Two porosity (θ_s) values are given: total and effective. Effective porosity accounts for trapped air and is the more reasonable value to use in computations. It can be seen in Table 1.4 that as the soil particles get finer, from sands to clays, the saturated hydraulic conductivity, K_s, decreases, the average wetting front suction, ψ_f, increases (negatively), and porosity, θ_s, is variable. Table 1.4 provides valuable estimates for Green-Ampt parameters, but local data (e.g., Carlisle et al., 1981) are preferable if available. Missing is the initial moisture content, θ_i, since it depends on antecedent rainfall and moisture conditions. Typical values for $M_d = \theta_s - \theta_i$ are given in the SCS Soil Properties and Interpretation sheets and are usually termed "available water (or moisture) capacity, in./in." Values usually range from 0.03 to 0.30. The value to use for a particular soil in question must be determined from a soil test. Otherwise, a conservative (low) M_d value could be used for design purposes (e.g., 0.10).

In areas of high water tables, there is a limit to the soil storage capacity and infiltration cannot continue indefinitely without complete saturation of the soil. In such cases, infiltration ceases, losses (rainfall abstractions) become zero, and rainfall excess intensity equals rainfall intensity. If site-specific information is available, this capacity, S, can be estimated from soil moisture data and depth to water table, L, as implied in Fig. 1.17(b),

$$S = L(\theta_s - \theta_i) \tag{1.31}$$

where L is now the depth to the water table. In some localities, regional information on available soil storage has been prepared (e.g., South Florida Water Management District, 1987).

EXAMPLE 1.8a

GREEN AND AMPT INFILTRATION EQUATION

For the following soil properties, develop a plot of infiltration rate f vs. infiltration volume F using the Green and Ampt equation:

$K_s = 1.97$ in./hr,
$\theta_s = 0.518$,
$\theta_i = 0.318$,
$\psi_f - 9.37$ in.,
$i = 7.88$ in./hr.

SOLUTION

Noting that $M_d = \theta_s - \theta_i$, we can solve Eq. (1.30) to obtain the volume of water that will infiltrate before surface saturation is reached:

$$F_s = \frac{\psi_f M_d}{(1 - i/K_s)}$$

$$= \frac{-(9.37 \text{ in.})(0.518 - 0.318)}{1 - [(7.88 \text{ in./hr})/(1.97 \text{ in./hr})]}$$

$$\boxed{F_s = 0.625 \text{ in.}}$$

Until 0.625 in. has infiltrated, the rate of infiltration is equal to the rainfall rate. After that point (surface saturation) the rate of infiltration is given by the equation (Eq. 1.29)

$$f = K_s(1 - M_d \psi_f / F)$$

Solving this equation for various values of F gives the graph shown in Fig. E1.8a, where f decreases as F increases.

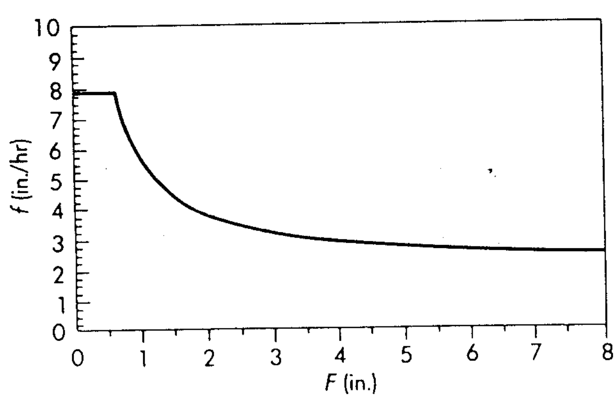

FIGURE E1.8(a)

EXAMPLE 1.8b

GREEN-AMPT TIME TO SURFACE SATURATION

Guelph loam has the following soil properties (Mein and Larson, 1973) for use in the Green-Ampt equation:

$K_s = 3.67 \times 10^{-4}$ cm/sec

$\theta_s = 0.523$

$\psi_f = -31.4$ cm water

For an initial moisture content of $\theta_i = 0.3$, compute the time to surface saturation for the following storm rainfall:

$i = 6K_s$ for 10 min

$i = 3K_s$ thereafter.

SOLUTION

The initial moisture deficit, $M_d = 0.523 - 0.300 = 0.223$. For the first rainfall segment, we compute the volume of infiltration required to produce saturation from Eq. (1.30):

$$F_s = \psi_f M_d/(1 - i/K_s) = (-31.4 \text{ cm})(0.223)/(1 - 6K_s/K_s) = 1.40 \text{ cm}.$$

The rainfall volume during the first 10 minutes is

$$10i = (10 \text{ min})(6 \times 3.67 \times 10^{-4} \text{ cm/sec})(60 \text{ sec/min}) = 1.31 \text{ cm}.$$

Since $1.31 < 1.40$, all rainfall infiltrates and surface saturation is not reached, and $F(10 \text{ min}) = 1.31$ cm.

The volume required for surface saturation during the lower rainfall rate of $i = 3K_s$ is

$$F_s = (-31.4 \text{ cm})(0.223)/(1 - 3K_s/K_s) = 3.50 \text{ cm}.$$

Thus, an incremental volume of $\Delta F = F_s - F(10 \text{ min}) = 3.50 - 1.31 = 2.19$ cm must be supplied before surface saturation occurs. This requires an incremental time of

$$\Delta t = \Delta F/i = (2.19 \text{ cm})/(3 \times 3.67 \times 10^{-4} \text{ cm/sec}) = 1989 \text{ sec} = 33.15 \text{ min}.$$

Thus, the total time to surface saturation is $10 + 33.15 = 43.15$ min.

1.6 STREAMFLOW

When rainfall strikes the land surface, it may initially distribute to fill depression storage, infiltrate to fill soil moisture and ground water, or travel as interflow to a receiving stream. Depression storage capacity is